Merton's model, credit risk and volatility skews

John C. Hull
Joseph L. Rotman School of Management, University of Toronto, 105 St George Street, Toronto, Ontario, Canada M5S 3E6; email: hull@rotman.utoronto.ca

Izzy Nelken
Super Computing Consulting Group, 3943 Bordeaux Drive, Northbrook, IL 60062-2135 USA; email: izzy@supercc.com

Alan D. White
Joseph L. Rotman School of Management, University of Toronto, 105 St George Street, Toronto, Ontario, Canada M5S 3E6; email: awhite@rotman.utoronto.ca

In 1974 Robert Merton proposed a model for assessing the credit risk of a company by characterizing the company's equity as a call option on its assets. In this paper we propose a method for estimating the model's parameters from the implied volatilities of options on the company's equity. We use data from the credit default swap market to compare our implementation of Merton's model with the traditional approach to implementation.

1 INTRODUCTION

The assessment of credit risk has always been important to banks and other financial institutions. Recently, banks have devoted even more resources than usual to this task. This is because, under the proposals in Basel II, regulatory credit risk capital may be determined using a bank's internal assessments of the probabilities that its counterparties will default.

One popular approach to assessing credit risk involves Merton's (1974) model. This model assumes that a company has a certain amount of zero-coupon debt that will become due at a future time T. The company defaults if the value of its assets is less than the promised debt repayment at time T. The equity of the

We are grateful to Moody's Investors Service for financial support and to GFI for making their data on credit default swap spreads available to us. We are also grateful to Jeff Bohn, Richard Cantor, Darrell Duffie, David Heath, Stephen Figlewski, Jerry Fons, David Lando, William Perraudin, Mirela Predescu, David Shimko (a referee), Roger Stein, Kenneth Singleton and workshop participants at Queens University, University of Toronto, York University, the Moody's Academic Advisory Committee, an International Association of Financial Engineers event, an International Monetary Fund event, and the Risk Management Conference of the Chicago Board Options Exchange for helpful comments on earlier drafts of this paper. Needless to say we are fully responsible for the content of the paper.

company is a European call option on the assets of the company with maturity T and a strike price equal to the face value of the debt. The model can be used to estimate either the risk-neutral probability that the company will default or the credit spread on the debt.[1]

As inputs, Merton's model requires the current value of the company's assets, the volatility of the company's assets, the outstanding debt and the debt maturity. One popular way of implementing his model estimates the current value of the company's assets and the volatility of the assets from the market value of the company's equity and the equity's instantaneous volatility using an approach suggested by Jones, Mason and Rosenfeld (1984). A debt maturity date is chosen and debt payments are mapped on to a single payment on the debt maturity date in some way.

In this paper we develop a new way of implementing Merton's model. This is based on use of the implied volatilities of options on the company's stock to estimate model parameters. Our approach is interesting both because it provides an alternative to Jones, Mason and Rosenfeld (1984) and because it gives insights into the linkages between credit markets and options markets.

Under Merton's model an option on the equity of a company is a compound option on the company's assets. Geske (1979), who provides a valuation formula for compound options, also shows that Merton's model is consistent with the type of volatility skew observed in equity markets.[2] In this paper we carry Geske's analysis one stage further to show that the credit spread in Merton's model can be calculated from the implied volatilities of two equity options. The options we choose are two-month at-the-money and out-of-the money put options.

To test our implementation of Merton's model and compare it with the more traditional approach to implementation we use credit default swap (CDS) spread data. A CDS is a derivative that protects the buyer against default by a particular company. The CDS spread is the amount paid for protection and is a direct market-based measure of the company's credit risk. Most previous researchers have used bond data to test implementations of Merton's model. Using CDS spreads is an attractive alternative. Bond prices have the disadvantage that they are often indications rather than firm quotes. Also, the credit spread calculated from a bond price depends on the bond's liquidity and involves an assumption about the benchmark risk-free rate.[3]

---

[1] A number of authors, such as Black and Cox (1976), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001), have developed interesting extensions of Merton's model, but none has emerged as clearly superior. See Eom, Helwege and Huang (2004), who compare the performance of alternative models using bond spreads. Gemmill (2002) shows that Merton's model works well in the particular case where zero-coupon bonds are used for funding.

[2] As the strike price of an equity option increases its volatility decreases. See Rubinstein (1994) and Jackwerth and Rubinstein (1996) for a discussion of this.

[3] A counterargument here is that the CDS market is not as well developed as the bond market. Players sometimes come to the market seeking unidirectional execution rather than asking for a bid and an offer.

The rest of this paper is organized as follows. Section 2 develops the theory that underlies our implementation of Merton's model. Section 3 describes the data we use. In Section 4 we compare the credit spreads implied by Merton's model with CDS spreads for both our implementation of Merton's model and the traditional implementation. In Section 5 we present some results on the theoretical relationships between implied volatilities and credit spreads under Merton's model and test whether these relationships hold. In Section 6 we develop a relatively simple model, based on Merton's (1976) jump–diffusion model, for relating credit spreads to implied volatilities and use it as a benchmark to test whether the more elaborate structure underlying our implementation of Merton (1974) provides a better explanation of observed credit spreads. Conclusions are presented in Section 7.

2 MERTON'S MODEL

Both Merton (1974) and Black and Scholes (1973) propose a simple model of the firm that provides a way of relating credit risk to the capital structure of the firm. In this model the value of the firm's assets is assumed to obey a lognormal diffusion process with a constant volatility. The firm has issued two classes of securities: equity and debt. The equity receives no dividends. The debt is a pure discount bond where a payment of D is promised at time T.

If at time T the firm's asset value exceeds the promised payment, D, the lenders are paid the promised amount and the shareholders receive the residual asset value. If the asset value is less than the promised payment, the firm defaults, the lenders receive a payment equal to the asset value, and the shareholders get nothing.

2.1 Equity value and the probability of default

Define E as the value of the firm's equity and A as the value of its assets. Let E_0 and A_0 be the values of E and A today and let E_T and A_T be their values at time T. In the Merton framework the payment to the shareholders at time T is given by

$$E_T = \max\left[A_T - D, 0\right]$$

This shows that the equity is a call option on the assets of the firm with strike price equal to the promised debt payment. The current equity price is therefore

$$E_0 = A_0 N(d_1) - D\mathrm{e}^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln\left(A_0 \mathrm{e}^{rT}/D\right)}{\sigma_A \sqrt{T}} + 0.5\sigma_A\sqrt{T}\,; \quad d_2 = d_1 - \sigma_A\sqrt{T}$$

σ_A is the volatility of the asset value and r is the risk-free rate of interest, both of which are assumed to be constant. Define $D^* = D\mathrm{e}^{-rT}$ as the present value of the promised debt payment and let $L = D^*/A_0$ be a measure of leverage. Using these definitions the equity value is

$$E_0 = A_0\left[N(d_1) - LN(d_2)\right] \tag{1}$$

where

$$d_1 = \frac{-\ln(L)}{\sigma_A\sqrt{T}} + 0.5\sigma_A\sqrt{T}; \quad d_2 = d_1 - \sigma_A\sqrt{T}$$

As shown by Jones, Mason and Rosenfeld (1984), because the equity value is a function of the asset value, we can use Itô's lemma to determine the instantaneous volatility of the equity from the asset volatility:[4]

$$E_0\sigma_E = \frac{\partial E}{\partial A}A_0\sigma_A$$

where σ_E is the instantaneous volatility of the company's equity at time zero. From Equation (1), this leads to

$$\sigma_E = \frac{\sigma_A N(d_1)}{N(d_1) - LN(d_2)} \tag{2}$$

Equations (1) and (2) allow A_0 and σ_A to be obtained from E_0, σ_E, L and T.[5] The risk-neutral probability, P, that the company will default by time T is the probability that shareholders will not exercise their call option to buy the assets of the company for D at time T. It is given by

$$P = N(-d_2) \tag{3}$$

This depends only on the leverage, L, the asset volatility, σ_A, and the time to repayment, T.

2.2 Debt value and the implied credit spread of risky debt

Merton's model can be used to explain risky debt yields. Define B_0 as the market price of the debt at time zero. The value of the assets at any time equals the total value of the two sources of financing, so that

$$B_0 = A_0 - E_0$$

[4] Jones, Mason and Rosenfeld (1984) actually use Equations (1) and (2) in conjunction with some estimates of A and s.

[5] The implementation of Merton's model, based on Equations (1) and (2), has received considerable commercial attention in recent years. Moody's KMV uses it to estimate relative probabilities of default. See Kealhofer (2003a, 2003b). CreditGrades (a venture supported by Deutsche Bank, Goldman Sachs, JP Morgan and the RiskMetrics Group) uses it to estimate credit default swap spreads. CreditGrades performs empirical tests similar to those we carry out for the traditional Merton model. See Finger (2002).

Using Equation (1), this becomes

$$B_0 = A_0\left[N(-d_1) + LN(d_2)\right] \tag{4}$$

The yield to maturity on the debt is defined implicitly by

$$B_0 = De^{-yT} = D^* e^{(r-y)T}$$

Substituting this into Equation (4) and using $A_0 = D^*/L$ gives the yield to maturity as

$$y = r - \ln\frac{\left[N(d_2) + N(-d_1)/L\right]}{T}$$

The credit spread implied by the Merton model is therefore[6]

$$s = y - r = -\ln\frac{\left[N(d_2) + N(-d_1)/L\right]}{T} \tag{5}$$

Like the expression for the risk-neutral probability of default in Equation (3), the implied credit spread depends only on the leverage, L, the asset volatility, σ_A, and the time to repayment, T.

2.3 Equity volatility and volatility skews

One point about Merton's model that has not received much exploration is the role it plays in explaining the equity option implied volatilities and the volatility skews that are observed in the equity options market. Within the framework of the Merton model, an option on the firm's equity that expires before the debt matures is a compound option, an option on a European call option. We can therefore use the model proposed by Geske (1979). Using the notation developed above, the value at time zero of a European put with strike price K and expiry time $t < T$ on the equity is

$$p = De^{-rT}M\left(-a_2, d_2; -\sqrt{\frac{\tau}{T}}\right) - A_0 M\left(-a_1, d_1; -\sqrt{\frac{\tau}{T}}\right) + Ke^{-r\tau}N\left(-a_2\right) \tag{6}$$

where

$$a_1 = \frac{\ln\left(A_0 / \mathrm{A}^*_\tau e^{-r\tau}\right)}{\sigma_A\sqrt{\tau}} + 0.5\,\sigma_A\sqrt{\tau}\,; \qquad a_2 = a_1 - 0.5\,\sigma_A\sqrt{\tau}$$

M is the cumulative bivariate normal distribution function and A^*_τ is the critical asset value at time τ, the value for which the equity value at that time equals K. That is, A^*_τ is the asset value below which the put on the equity will be exercised.

[6] The relationship between credit spreads and default risk is discussed by Duffie and Singleton (1999), Litterman and Iben (1991) and Rodriguez (1988) among others.

Define v as the implied volatility of the put at time zero based on the Black–Scholes model. Also define parameters α and κ by

$$A_\tau^* = \alpha A_0 \mathrm{e}^{r\tau}, \qquad K = \kappa E_0 \mathrm{e}^{r\tau}$$

The parameter α is the ratio of the critical asset price to the forward asset price (both being observed at time zero). We will refer to it as the *implied strike level*. The parameter κ is the ratio of the option strike price to the forward equity price (observed at time zero). We will refer to it as the option's *moneyness*.

Implied volatilities are the volatilities which, when substituted into the Black–Scholes model, give the market price. If we assume that market prices are given by Merton's model, the implied volatility of an option can be determined by solving

$$D^* M\left(-a_2, d_2; -\sqrt{\frac{\tau}{T}}\right) - A_0 M\left(-a_1, d_1; -\sqrt{\frac{\tau}{T}}\right) + \kappa E_0 N(-a_2)$$
$$= \kappa E_0 N(-d_2^*) - E_0 N(-d_1^*) \tag{7}$$

where

$$d_1^* = \frac{-\ln(\kappa)}{v\sqrt{\tau}} + 0.5 v\sqrt{\tau}\,; \qquad d_2^* = d_1^* - v\sqrt{\tau}$$

$$a_1 = \frac{-\ln(\alpha)}{\sigma_A\sqrt{\tau}} + 0.5\sigma_A\sqrt{\tau}\,; \qquad a_2 = a_1 - \sigma_A\sqrt{\tau}$$

Substituting Equation (1) into Equation (7) results in

$$LM\left(-a_2, d_2; -\sqrt{\frac{\tau}{T}}\right) - M\left(-a_1, d_1; -\sqrt{\frac{\tau}{T}}\right) + \kappa N(-a_2)[N(d_1) - LN(d_2)]$$
$$= [\kappa N(-d_2^*) - N(-d_1^*)][N(d_1) - LN(d_2)] \tag{8}$$

A variation of Equation (1) can be used to determine the implied strike level, α:

$$\kappa E_0 \mathrm{e}^{r\tau} = A_\tau^*[N(d_{1,\tau}) - (L/\alpha)N(d_{2,\tau})]$$

so that

$$\kappa = \frac{\alpha N(d_{1,\tau}) - LN(d_{2,\tau})}{N(d_1) - LN(d_2)} \tag{9}$$

where

$$d_{1,\tau} = \frac{-\ln(L/\alpha)}{\sigma_A\sqrt{T-\tau}} + 0.5\sigma_A\sqrt{T-\tau}\,; \qquad d_{2,\tau} = d_{1,\tau} - \sigma_A\sqrt{T-\tau}$$

Equations (8) and (9) define an implicit relationship between the implied volatility of an option and the moneyness, κ, for a set of model parameter values L, σ_A, and T, and the option maturity, τ. For different values of κ different implied volatilities, v, will result, leading to a volatility skew. For all values of the model parameters, the implied volatilities are of the form observed in practice where an increase in the strike price leads to a reduction in the implied volatility.

2.4 An alternative implementation of Merton's model

Section 2.3 suggests a new way of implementing Merton's model using two implied volatilities. With one implied volatility we can solve Equations (8) and (9) for a particular value of T to obtain a relationship between the leverage ratio, L, and the asset volatility, σ_A. With two implied volatilities we have two such relationships that can be solved for L and σ_A. The credit spread for a zero-coupon bond maturing at time T can then be calculated using Equation (5).

This implementation approach allows credit spreads to be estimated directly from implied volatility data. It is a potentially attractive alternative to the traditional implementation based on Equations (1) and (2) because it avoids the need to estimate the instantaneous equity volatility and the need to map the company's liability structure (some of which may be off balance sheet) on to a single zero-coupon bond.

In the sections that follow we will compare the results obtained from our implementation of Merton's model with the traditional implementation. As a benchmark we will also examine the performance of a simpler model where a Poisson process generates defaults.

3 DATA

Our empirical tests are based on credit default swap data, implied volatility data, equity price data and balance sheet data for companies between January and December, 2002.

3.1 Credit default swap data

A credit default swap or CDS provides insurance against a default by a particular company or sovereign entity. The company is known as the *reference entity* and a default by the company is known as a *credit event*. The buyer of the insurance makes periodic payments to the seller and in return obtains the right to sell a bond issued by the reference entity for its face value if a credit event occurs. The amount of the payments made per year by the buyer is known as the *CDS spread*.[7,8] The credit default swap market has grown rapidly since the International Swaps

[7] In a standard contract, payments by the buyer are made quarterly or semiannually in arrears. If the reference entity defaults, there is a final accrual payment and payments then stop. Contracts are sometimes settled in cash rather than by the delivery of bonds. In this case there is a calculation agent who has the responsibility of determining the (*continued on next page*)

and Derivatives Association produced its first version of a standardized contract in 1998.[9]

GFI, a broker specializing in the trading of credit derivatives, provided us with CDS quotes for the period January to December, 2002. The data contain in excess of 120,000 individual CDS quotes. Each quote contains the following information:

- the date on which the quote was made;
- the name of the reference entity;
- the life of the CDS;
- whether the quote is a bid (wanting to buy protection) or an offer (wanting to sell protection); and
- the CDS spread quote in basis points.

Each quote is a firm quote for a minimum notional of US$10 million. The reference entity may be a corporation such as Blockbuster Inc., a sovereign such as Japan, or a quasi-sovereign such as the Federal Home Loan Mortgage Corporation. During the period we are considering CDS quotes are provided on 1,597 named entities: 1,500 corporations, 60 sovereigns and 37 quasi-sovereigns. Of the reference entities 796 are North American, 451 are European and 330 are Asian and Australian. The remaining reference entities are African or South American.

We used only five-year quotes in our analysis (approximately 85% of the quotes in our data set were for five-year CDS). When there were both five-year bid and offer quotes on a reference entity in a day, we calculated what we will refer to as an "observation" on the five-year CDS spread as the average of the maximum bid and minimum offer.[10] As shown by Duffie (1999) and Hull and White (2000), the five-year credit default swap spread is, in theory, very close to the credit spread of the yield on a five-year par yield bond issued by the reference entity over five-year par yield risk-free rate.

3.2 Implied volatility data

From the list of entities covered by the CDS data, we chose optionable US equities with current stock prices over US$20. This resulted in a list of 325 companies. The Bloomberg system, which has been archiving the implied volatility of options on US equities since the beginning of 2002, was polled to determine for which of the names implied volatility data were available. This reduced the

market price, x, of a bond issued by the reference entity a specified number of days after the credit event. The payment by the seller is then is $100 - x$ per $100 of principal.

[8] The CDS spread is very close to the credit spread observed in the corporate bond market when the credit spread is measured relative to the swap rate. For more information on the relation between CDS spreads and bond credit spreads, see Blanco, Brennan and Marsh (2003), Longstaff, Mithal and Neis (2003) or Hull, Predescu and White (2004).

[9] A more complete description of credit default swaps and the CDS market can be found in, for example, Duffie (1999).

[10] When there was a trade the bid equals the offer.

sample size to 319 firms. For this sample, the implied volatility for the two-month 50-delta put and the 25-delta put were downloaded for every trading day in the year 2002. This produced 61,544 observations.

In the Bloomberg system a two-month option is defined as an option with a maturity of between one and two months. In most circumstances the exact maturity date of the option can be calculated from knowledge of the maturity dates of options traded on the Chicago Board Option Exchange. The two-month 25-delta put implied volatility is an estimate of the implied volatility of a two-month put option that has a delta of -0.25. This is calculated by interpolating between the implied volatility of the two two-month options whose deltas are closest to -0.25, one greater than -0.25 and the other less than -0.25. The implied volatility for a two-month 50-delta put is calculated similarly. We chose the 50-delta and 25-delta implied volatilities because they are usually calculated from relatively liquid options.[11]

3.3 Data for traditional implementation

For the traditional approach to implementing Merton's model we downloaded daily closing stock prices and quarterly balance sheet information from Bloomberg for the period between January 1, 2001, and December 31, 2002. We used the reported quarterly total liabilities divided by the quarterly reported outstanding number of shares as our measure of the debt claim, D. On any particular day the current value of the debt claim was set to the most recently reported value. The equity price and the instantaneous equity volatility used in Equations (1) and (2) were the daily closing price and a historic volatility estimated using the most recent 40 returns. The choice of 40 business days to estimate volatility is a trade-off. Using a longer (shorter) period to estimate volatility results in more (less) accurate but less (more) timely estimates. The total number of observations was 63,627.

3.4 Merged data

We merged the volatility data and the data used in the traditional implementation with the CDS data. This combined data set was then filtered to eliminate cases in which the volatility exceeded 90% (65 cases) and firms for which there were 10 or fewer implied volatility observations or CDS quotes. This resulted in a pool of 127 firms, each with between 11 and 178 days of data for a total of 6,220 firm days of data. In other words we ended up with 6,220 cases where, for a particular company on a particular day, we had (1) a CDS spread observation, (2) a two-month 50-delta put implied volatility, (3) a two-month 25-delta put implied volatility, (4) an equity value, (5) a debt claim, and (6) a historic equity volatility estimate. These are the data that were used for the analysis described in the following sections.

[11] Bloomberg implied volatilities are sometimes criticized for the way they handle items such as dividends, bid–offer spreads and American exercise features. Our objective is to test the ability of the model to rank order credit spreads. Any consistent biases in Bloomberg's implied volatility estimates may not therefore be important.

4 EMPIRICAL TESTS

In this section we test whether five-year credit spreads implied from our implementation of Merton's model and the traditional implementation are consistent with the observed five-year CDS spreads. For the traditional implementation we estimated the equity value, historical equity volatility and the outstanding debt in the way described in Section 3.3. Equations (1) and (2) were used to compute the asset value and asset volatility and Equation (5) was then used to compute the credit spread. For our implementation we used the 50- and 25-delta implied put volatilities in Equations (8) and (9) to imply the leverage ratio and the asset volatility.[12] As in the case of the traditional implementation, Equation (5) is then used to compute the credit spread. In the balance of the discussion we shall refer to our implementation of Merton's model as the "ImpVol" implementation and the traditional implementation as the "Trad" implementation.

There are a number of reasons why we should expect differences between the credit spreads implied from Merton's model and observed CDS spreads. Merton's model is not a perfect representation of reality because companies do not usually issue only zero-coupon debt and because a number of factors besides the value of their assets are liable to influence a company's decision to default on its obligations. Also, CDS spreads are likely to be slightly different from bond yield spreads for the reasons listed in Hull, Predescu and White (2004). Finally, the credit spread backed out from Merton's model is the spread between the yields on zero-coupon bonds, while a CDS credit spread is (at least, approximately) the spread between the yields on par yield bonds.

Table 1 shows the results of regressing CDS spreads against the spreads implied from Merton's model using the two implementation approaches. Given the nature of the model generating the implied spreads, it is unlikely that errors in the implied spreads are normally distributed. However, the regression does provide a first attempt at describing the relationship (if any) between the implied and observed spreads.

The results in Table 1 reveal a positive relationship between the observed CDS spreads and the implied spreads that is roughly similar for both implementations. The mean CDS observed spread is about 95 basis points higher than the mean implied spread for both models. The R^2 of the regressions indicate that the ImpVol implementation provides a better fit to the observed data than the Trad implementation.

It is possible that there are factors other than those suggested by Merton's model that affect CDS spreads. Figure 1 shows a scatter diagram of the CDS spread versus the implied spread from the ImpVol model for Merrill Lynch, Dow Chemicals and Bowater. Figure 2 shows the same for the Trad model. These figures suggest that the relation between the CDS spread and the implied spread may be different for different firms. It is also possible that macroeconomic variables cause the relationship between the CDS spread and the implied spread

[12] This involves determining the value of κ that produces options with deltas of 0.50 and 0.25.

TABLE 1 Regression of CDS spreads against implied credit spreads.

Implementation	Constant	Slope	R^2	n
ImpVol	92.564	0.307	0.178	6,220
	(1.017)	(0.008)		
Trad	95.811	0.157	0.069	6,220
	(1.137)	(0.007)		

The credit spreads are implied from the ImpVol implementation and the Trad implementation of Merton's (1974) model. The ImpVol implementation is the implementation we propose in Equations (8) and (9). The Trad implementation is the traditional implementation of Merton's model in Equations (1) and (2). Standard errors are shown in parentheses.

TABLE 2 Regression of CDS spreads against implied credit spreads firm by firm and day by day.

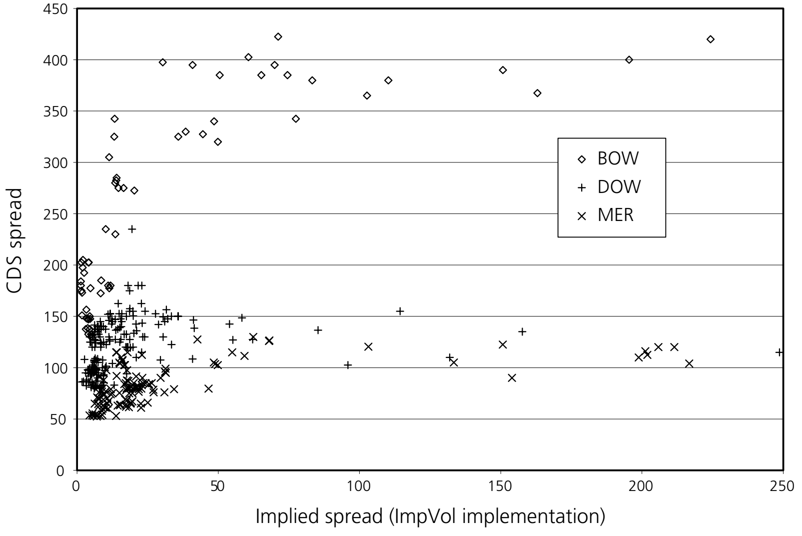
Implementation		Constant	Slope	R^2	n
		Time series by ticker			
ImpVol	Mean	100.55	0.33	0.26	63
	Median	76.17	0.15	0.23	53
Trad	Mean	96.31	0.22	0.29	63
	Median	79.53	0.14	0.28	53
		Cross-section by day			
ImpVol	Mean	90.95	0.88	0.24	37
	Median	92.68	0.36	0.18	35
Trad	Mean	97.17	0.40	0.12	37
	Median	99.37	0.21	0.06	35

Mean and median results for regressions of observed CDS spread against implied credit spread. The credit spreads are implied from the ImpVol implementation and the Trad implementation of Merton's (1974) model. The ImpVol implementation is that proposed in Equations (8) and (9). The Trad implementation is the traditional implementation of Merton's model in Equations (1) and (2). The upper panel gives results for time-series regressions done on a firm-by-firm basis (86 regressions), and the lower panel those for cross-sectional regressions done on a day-by-day basis (90 regressions). Only regressions with 30 or more observations are included.

to change through time. To explore these possibilities we carried out a separate regression for each firm and a separate regression for each day.

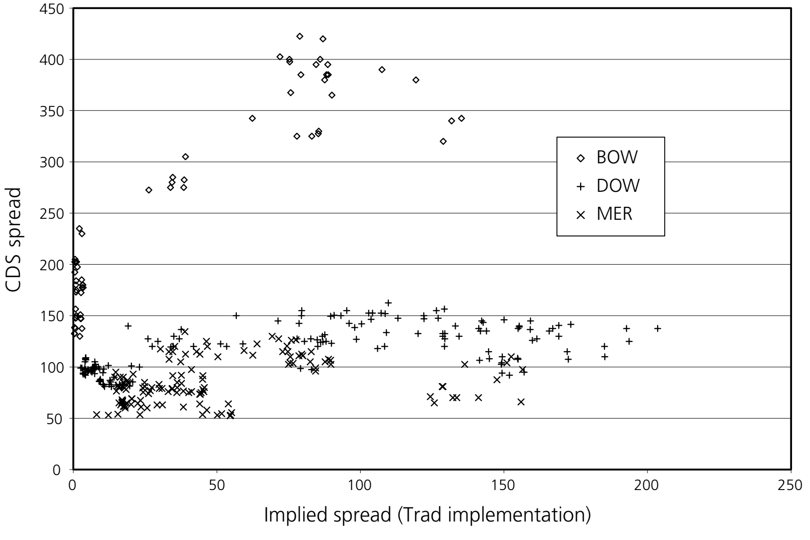
The results are shown in Table 2. For both the firm-by-firm regressions and the day-by-day regressions we report mean and median values for the constant, slope, R^2, and number of observations. Any firm for which there were less than 30 observations was not included in the firm-by-firm regressions. Any day for which there were less than 30 observations was not included in the day-by-by regression. This resulted in 86 firm-by-firm regressions and 90 day-by-day regressions. Table 2 shows that, on average, the implied spreads fit the CDS observed spreads much better when considered on a firm-by-firm or day-by-day basis than when fitting the entire sample. This is not surprising since the number of degrees of freedom is much larger in the firm-by-firm or day-by-day analysis.

FIGURE 1 Relationship between CDS spread and implied spread for ImpVol implementation of Merton's model.



The three companies are Merrill Lynch (MER), Bowater (BOW) and Dow Chemicals (DOW). The ImpVol implementation of Merton's model is the implementation we propose in Equations (8) and (9).

FIGURE 2 Relationship between CDS spread and implied spread for Trad implementation of Merton's model.



The three firms are Merrill Lynch (MER), Bowater (BOW) and Dow Chemicals (DOW). The Trad implementation is the traditional implementation in Equations (1) and (2).

On the basis of the regression R^2, the results also show that the implied spreads match CDS spreads better on a firm-by-firm basis than when looking across firms on a particular day. What this means is that the models seem to work better at explaining how the observed credit spreads for a firm change over time than they do at discriminating between different firms at a single time. The two implementations are comparable when applied on a firm-by-firm basis, but the ImpVol implementation appears to provide a better fit than the Trad implementation when applied cross-sectionally.

4.1 Rank correlations

The results presented so far indicate that both implementations of Merton's model are consistent with the data in the sense that there is a positive relationship between the model predictions and the observed data. There is also some evidence that the relation between the implied credit spread and observed credit spread is non-linear.

To address the apparent non-linearity we could test alternative non-linear models. However, the nature of the non-linearity is not known and it may differ from firm to firm. A general approach to fitting data that are subject to an unknown non-linear relationship is to linearize the data by translating observations to ranks. Formally, if

$$y = f(x)$$

for some monotonic increasing function, f, and we have a set of observations $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$, then

$$r\big(y_i \,\big|\, \{y_1, y_2, \ldots, y_n\}\big) = r\big(x_i \,\big|\, \{x_1, x_2, \ldots, x_n\}\big)$$

where $r(a \mid \{b\})$ is the rank of a within the set b. This linearization works perfectly in the deterministic case described here, but difficulties arise when variables are observed with error. However, there is a well-developed literature on rank correlation which will allow a more formal and robust distribution-free test to determine which version of the model is more consistent with the data.[13]

There are two measures of rank order correlation in the literature: Kendall's and Spearman's. To explain how they are calculated, suppose that we have n observations on two variables. (In our application the variables are five-year implied credit spread and five-year observed CDS spread). As a first step we calculate the rank of each observation on each variable.

[13] In the context of our tests, it is interesting to note that proponents of the commercial use of Merton's model claim that, although estimated default probabilities and credit spreads are not accurate, the models rank the credit quality of companies well. See, for example, Kealhofer (2003a, 2003b).

For the Kendall rank order correlation measure, r_k, we look at the $n(n-1)/2$ alternative pairs of observations. If the rankings of variables for a particular pair of observations are in the same order we score +1, and if they are in inverse order we score −1. For example, if for a particular pair of observations the rankings of the first variable were 5 and 10, respectively, and the rankings of the second variable were 6 and 8, respectively, we would score +1. If the rankings of the first variable were 5 and 10, respectively, and the rankings of the second variable were 8 and 6, respectively, we would score −1. The rank order correlation is the sum of the scores for all pairs divided by $n(n-1)/2$.

For the Spearman's rank order correlation, r_s, we calculate for each observation i the difference, d_i, between the rank order of the first variable and the rank order of the second variable. The correlation measure is

$$r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n^3 - n}$$

Kendall and Gibbons (1990) provide a great deal of information on the properties of the two correlation measures. The properties we will use and the statistical tests they give rise to are outlined in the appendix.

The formal statistical tests based on the rank order correlation between implied spreads and observed CDS spreads are reported in Table 3. The first panel in Table 3 shows the results from pooling all the data. The middle panel presents a firm-by-firm analysis. Rank order correlations were calculated for every company that had 30 or more observations, as in Table 2. The final panel in Table 3 tests whether Merton's model can be used to rank the relative credit quality of different firms at a point in time. Rank correlations were computed for every day on which data were available for 30 or more firms, as in Table 2.

The broad conclusions from the results in Table 3 are similar to those from the results in Tables 1 and 2. The z-statistics show that we can reject the null hypothesis that the rank order correlations are zero with a very high degree of confidence in all cases for both implementations of Merton's model. The rank order correlations within firms across time are always higher than those measured within a day across firms.[14] This indicates that both implementations do better at tracking a single firm over time than they do at distinguishing between firms at a point in time.

The correlations from the ImpVol implementation of Merton's model are always higher than for the Trad implementation. In the case of the firm-by-firm analysis they are significantly higher at the 1% level. This is in contrast to the results in Table 2, where the R^2 for Trad is slightly higher than the R^2 for ImpVol in the firm-by-firm case. A close inspection revealed that the R^2 for the firm-by-case case in Table 2 was greatly influenced by a few outliers. Outliers have far less effect on rank order correlations.

[14] Although the calculations are not shown, in every case the within-firm rank order correlation is significantly larger than the corresponding within-day rank order correlation at the 1% level.

TABLE 3 Rank order correlation measures.

	Kendall rank correlation	Standard error	*z*-statistic	Spearman rank correlation	Standard error	*z*-statistic
All data						
ImpVol	0.2836	0.0172	33.55	0.4230	0.0199	33.36
Trad	0.2590	0.0173	30.63	0.3929	0.0202	30.99
ImpVol – Trad	0.0247	0.0244	1.01	0.0301	0.0284	1.06
Firm-by-firm (n = 86)						
ImpVol	0.3967	0.0188	38.01	0.5409	0.0202	35.51
Trad	0.3101	0.0193	28.71	0.4386	0.0212	28.79
ImpVol – Trad	0.0866	0.0270	3.21	0.1023	0.0293	3.49
Day-by-day (n = 90)						
ImpVol	0.2506	0.0239	20.37	0.3630	0.0280	20.35
Trad	0.2188	0.0241	17.78	0.3186	0.0285	17.87
ImpVol – Trad	0.0318	0.0340	0.94	0.0443	0.0400	1.11

Table gives Kendall rank order correlation measure and Spearman rank order correlation measure between the implied credit spread from Merton's (1974) model and the five-year credit default swap spread. The ImpVol implementation is the implementation of Merton's model we propose in Equations (8) and (9). The Trad implementation is the traditional implementation of Merton's model in Equations (1) and (2). The standard error of the rank order correlation is an upper bound. The z-statistic tests whether the rank order correlation is significantly greater than zero. The A – B rows use Equation (A1) to test the difference between the A and B correlations.

There are a number of possible reasons why the ImpVol implementation ranks credit spreads better than the Trad implementation. Implied volatilities adjust to information more quickly than do historic volatilities. There is noise in the estimates of the historic volatility for the Trad implementation, and the Trad implementation requires the assumption that the historic volatility is the same as the instantaneous volatility. Also, we had only limited information on the company's capital structure for the Trad implementation.

4.2 Impact of debt maturity

As the debt maturity date in the ImpVol implementation of Merton's model changes the implied credit spread changes, but there is little effect on the ranking of implied credit spreads. This is illustrated in Table 4, which shows the rank order correlation between the implied credit spreads when T equals 1, 2, 5 and 10. We obtained similar results for the probability of default rankings and for the day-by-day and company-by-company analyses. For financial institutions that are interested only in ranking the creditworthiness of counterparties, this result may add to the attraction of the ImpVol implementation of Merton's model.

TABLE 4 Rank order correlations for different values of the debt maturity, *T*.

	***T* = 1**	***T* = 2**	***T* = 5**	***T* = 10**
***T* = 1**	1.000	0.978	0.954	0.988
***T* = 2**	0.878	1.000	0.946	0.974
***T* = 5**	0.826	0.800	1.000	0.947
***T* = 10**	0.908	0.861	0.809	1.000

Table gives Spearman and Kendall rank order correlations between the credit spreads implied from Merton's model for different values of the debt maturity, *T*, using the ImpVol implementation. The upper triangular portion of the matrix shows the Spearman correlation, the lower triangular portion shows the Kendall correlation.

5 PROPERTIES OF MERTON'S MODEL

In this section we show that Merton's model makes certain predictions about the nature of the relationship between implied volatilities and credit spreads and test whether the predictions are supported by the data. We define ATMVOL for maturity τ as the implied volatility for an option with a delta of 0.50. We define SKEW for maturity τ as the implied volatility for an option with maturity τ and a delta of 0.25 minus the implied volatility for an option with maturity τ and a delta of 0.50.

Our implementation of Merton's model can be used to relate T-year credit spreads to ATMVOL and SKEW for particular values of τ and T. Figure 3 shows the theoretical relationship between credit spread and ATMVOL for different values of SKEW when $T = 5$ and $\tau = 0.1667$. The figure shows that there is a pronounced positive relationship with positive convexity. Similar results are obtained for other values of T and τ. Figure 4 shows the relationship between credit spread and SKEW for different values of ATMVOL when $T = 5$ and $\tau = 0.1667$. There is very little relationship for low values of ATMVOL (volatility less than 50%) but a strongly positive relationship for higher levels of ATMVOL.

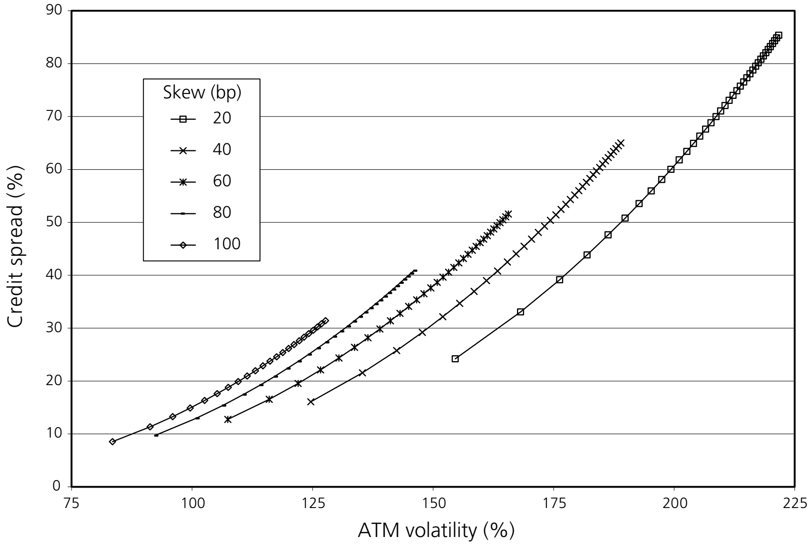
To test whether these properties of the model are supported by the data we performed a linear regression of observed CDS spread (CREDSPR) against ATMVOL and SKEW for our data:

$$CREDSPR = a + b \times ATMVOL + c \times SKEW$$

ATMVOL and SKEW are quite highly correlated. To address this co-linearity issue the regressions were done twice, once orthogonalizing SKEW with respect to ATMVOL and once orthogonalizing ATMVOL with respect to SKEW. To explore both the convexity of the relation between CREDSPR and ATM and the increasing slope of the relation between CREDSPR and SKEW for higher ATM volatilities, we partitioned the sample into subsets based on the ATM volatility.[15] The results are given in Table 5.

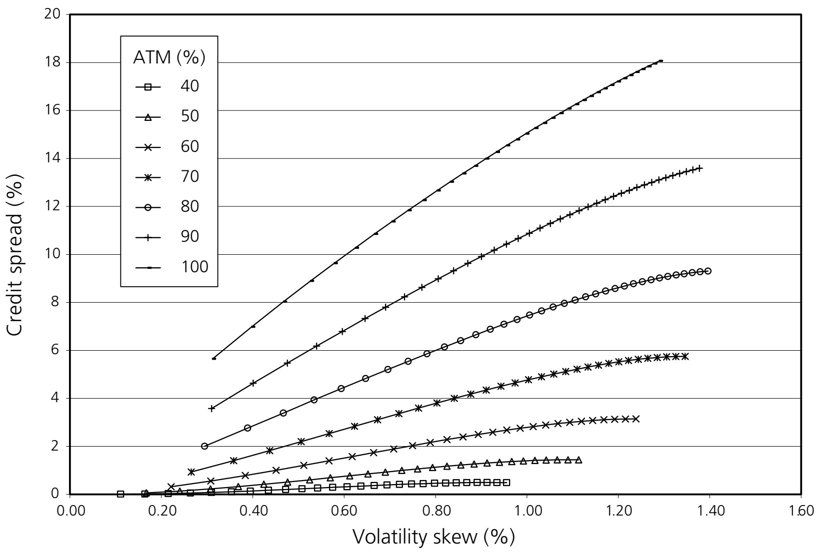
[15] The partitioning was based on the average ATM volatility for each firm. As a result, each firm appears in only one subset.

FIGURE 3 Theoretical relationship between credit spread and at-the-money volatility.



The relationship is implied by Merton's model for alternative values of the volatility skew when option maturity is two months and debt maturity is five years. The volatility skew is the difference between the volatility of an option with a delta of 0.25 and an option with a delta of 0.5.

FIGURE 4 Theoretical relationship between credit spread and volatility skew.



Relationship is implied by Merton's model for alternative at-the-money (ATM) volatilities when option maturity is two months and debt maturity is five years. The volatility skew is the difference between the volatility of an option with a delta of 0.25 and an option with a delta of 0.5.

TABLE 5 Regression of observed CDS spread against the at-the-money volatility and the volatility skew.

	Orthogonalized SKEW			Orthogonalized ATMVOL				
Restriction	***a***	***b***	***c***	***a***	***b***	***c***	R^2	***n***
None	−27.61*	336.47*	57.83	75.63*	328.26*	657.50*	0.1889	6,220
	(3.68)	(8.84)	(36.86)	(1.81)	(10.27)	(31.72)		
ATMVOL < 40%	7.62†	222.96*	14.13	72.31*	220.68*	313.74*	0.1318	3,088
	(3.72)	(10.28)	(33.75)	(1.58)	(11.63)	(29.82)		
40% < ATMVOL	−38.99*	371.91*	166.69†	86.42*	346.91*	794.51*	0.1413	3,132
	(7.62)	(16.45)	(66.95)	(3.48)	(19.28)	(57.15)		

Table presents results of a regression of observed CDS spread (CREDSPR) against the at-the-money volatility (ATMVOL) and the volatility skew (SKEW): $CREDSPR = a + b \times ATMVOL + c \times SKEW$. SKEW is measured as the 25-delta volatility less the at-the-money volatility. Volatilities are measured as decimals and the CDS spreads are in basis points. Standard errors are shown in parentheses. To address co-linearity issues, the regressions were done first with SKEW orthogonalized with respect to ATMVOL and then with ATMVOL orthogonalized with respect to SKEW.
*Denotes significance at the 1% level. †Denotes significance at the 5% level.

5.1 Relation between credit spread and ATM volatility

When SKEW is orthogonalized with respect to ATMVOL, Table 5 shows that, as predicted, there is a significantly positive slope for the ATMVOL for the full sample. When the sample is split into two roughly equal subsets based on ATMVOL, we find that the coefficient of the ATMVOL is consistently significantly positive. Further, the coefficient of the ATMVOL is higher in the high-volatility subset than it is in the low-volatility subset. This is consistent with the convex relationship predicted by the model. The increase in slope is statistically significant.

When ATMVOL is orthogonalized with respect to SKEW, some of the impact of the positive relation between CREDSPR and ATMVOL is liable to be captured by the SKEW coefficient. However, it is reassuring that the coefficient ATMVOL is still always significantly positive at the 1% level. Furthermore, it is significantly higher for the high-volatility subset than for the low-volatility subset. Table 5 therefore provides strong evidence for a convex positive relation between CREDSPR and ATMVOL.

5.2 Relation between credit spread and skew

When ATMVOL is orthogonalized with respect to SKEW the relation between CREDSPR and SKEW is always significantly positive at the 1% level. When the sample is split into two groups, the coefficient of SKEW is significantly higher in the high-volatility group than in the low-volatility group, and the difference is significant.

When SKEW is orthogonalized with respect to ATMVOL, the coefficient of SKEW is not significant overall. It is not significant for the lower-volatility subset and is significant at the 5% level for the high-volatility subset. This is consistent with the model, although less strongly so than the results in Section 5.1.

A general observation is that the volatility skews observed in practice are much higher than those that could reasonably be predicted by Merton's model. The reason may be what Rubinstein (1994) has referred to as "crash-o-phobia".

6 IS OUR VERSION OF MERTON'S MODEL OVER-STRUCTURED?

As shown by Equations (8) and (9), Merton's model provides a fairly complex relationship between implied volatilities and credit spreads. Some banks have instead tried much simpler linear or log-linear regression models of credit spreads against implied volatilities and other variables. An important question is whether Merton's model outperforms such simpler models. In this section we develop as a benchmark a model for relating credit spreads to implied volatilities with far less structure. We test whether Merton's (1974) model outperforms this benchmark.

In Merton's (1976) jump–diffusion model, the stock price obeys the process:

$$\frac{\mathrm{d}S}{S} = (\mu - \lambda k)\,\mathrm{d}t + \sigma_E \mathrm{d}z + \mathrm{d}q$$

where $\mathrm{d}q$ is a Poisson process with intensity λ and jump size k. The probability that a jump occurs in some small time interval, $\mathrm{d}t$, is $\lambda \mathrm{d}t$. In the event of a jump the change in the stock price is $\mathrm{d}S = kS$. The expected return on the stock is

$$E\frac{(\mathrm{d}S/S)}{\mathrm{d}t} = (\mu - \lambda k) + \lambda k = \mu$$

We consider the particular case of this model where $k = -1$. In this case jumps always lead to a zero stock price. We assume that a zero stock price coincides with a default. Defaults are therefore generated by a Poisson process with intensity λ.[16]

Consider a put option with strike price K and time to maturity τ. As before, we define a moneyness variable κ by

$$K = \kappa S_0 \mathrm{e}^{r\tau}$$

where S_0 is today's stock price. Merton showed that the option's price is

$$S_0\left[\kappa \mathrm{e}^{-\lambda\tau} N(-d_2) - N(-d_1) + \kappa\left(1 - \mathrm{e}^{-\lambda\tau}\right)\right] \tag{10}$$

where

$$d_1 = \frac{\lambda - \ln\kappa}{\sigma_E\sqrt{\tau}} + 0.5\sigma_E\sqrt{\tau}\,; \quad d_2 = d_1 - \sigma_E\sqrt{\tau}$$

[16] This is similar to reduced-form models such as those proposed by Duffie and Singleton (1999).

Equating this option price to the Black–Scholes price defines the implied volatility v:

$$\kappa e^{-\lambda\tau}N(-d_2) - N(-d_1) + \kappa(1 - e^{-\lambda\tau}) = \kappa N(-d_2^*) - N(-d_1^*) \tag{11}$$

where, as before,

$$d_1^* = \frac{-\ln(\kappa)}{v\sqrt{\tau}} + 0.5v\sqrt{\tau}; \quad d_2^* = d_1^* - v\sqrt{\tau}$$

If a volatility, σ_E, and a default intensity, λ, are chosen and Equation (11) is solved to determine the Black–Scholes implied volatility for various values of κ, a volatility skew results. Similarly to the case of the Merton model discussed in Section 2.3, the lower the value of κ, the higher is the implied volatility, v.[17]

Merton's (1976) model can also be used to determine the price of a zero-coupon bond issued by the firm that matures at time T. If no default occurs, the bond is assumed to pay \$1 at maturity, and if a default occurs at or before maturity, the bondholder is assumed to recover $R \le \$1$ at maturity.[18] Conditional on the intensity of the Poisson process, the risk-neutral probability that no default will occur before maturity, π, is $\exp[-\lambda T]$ and the probability that a default will occur is $1 - \pi$. The bond price is the present value of the risk-neutral expected value discounted back to the present using the risk free rate of interest.

$$[\pi + (1 - \pi)R]e^{-rT} = e^{-yT}$$

where y is the yield on the zero-coupon bond. The credit spread on the debt is then

$$y - r = \frac{-\ln[\pi + (1 - \pi)R]}{T}$$

or

$$\frac{-\ln[e^{-\lambda T} + (1 - e^{-\lambda T})R]}{T} \tag{12}$$

In the event that the recovery rate, R, is zero, the credit spread equals the default intensity, λ, for all maturities.

[17] A version of the jump–diffusion model in which the diffusion part of the process obeys a constant elasticity of variance process was also tested. The results from the CEV version were not materially different from the results given by the version described here and are not reported.
[18] This is equivalent to the assumption that the claim in the event of default is proportional to the default-risk-free value of the debt.

TABLE 6 Comparison of Merton's (1974) model with a model based on Merton's (1976) mixed jump–diffusion process.

	Kendall rank correlation	Standard error	*z*-statistic	Spearman rank correlation	Standard error	*z*-statistic
Merton (1974)	0.2836	0.0172	33.546	0.4230	0.0199	33.362
Merton (1976)	0.2095	0.0175	24.778	0.3177	0.0208	25.056
Merton (1974) – Merton (1976)	0.0741	0.0246	3.019	0.1053	0.0288	3.657

Data are pooled for all companies on all days. The table shows Kendall's rank order correlation measure and Spearman's rank order correlation measure between implied credit spread and the five-year credit default swap spread.

The standard error of the rank order correlation is an upper bound. The *z*-statistic tests whether the rank order correlation is significantly greater than zero. The last row uses Equation (A1) to test the difference between the Merton (1974) and Merton (1976) correlations.

Just as Equations (8), (9) and (5) allowed us to imply credit spreads from option volatilities under the Merton (1974) model, Equations (11) and (12) allow us to do so under the Merton (1976) model. Option implied volatilities can be used to infer the default intensity and the stock volatility, and these parameters can be used to imply a credit spread. As in Section 4, this can be compared with the contemporaneous CDS spread. In doing the analysis it is necessary to assume a time to debt maturity, T, and a recovery rate, R. The magnitude of the resulting implied credit spread is sensitive to these assumptions but the relative ranking of outcomes is not.

These results allow us to provide an interesting test of the value of the structural model underlying Merton (1974). Our null hypothesis is that Merton (1976) ranks the credit quality of companies as well as Merton (1974). Table 6 compares the performance of Merton's (1976) model with Merton's (1974) model. The 1976 model has statistically significant explanatory power, but in all cases the Merton (1974) model provides significantly better predictions of default probabilities and credit spreads at the 1% level.

7 CONCLUSION

The traditional approach to implementing Merton's model involves estimating the instantaneous equity volatility and the debt outstanding by a particular future time. We have presented an alternative implementation where the inputs to the model are much simpler. All that is required to imply a credit spread is two implied volatilities. The alternative approach is particularly appropriate for firms that are known (or rumored) to have significant off-balance-sheet liabilities.

Our proposed implementation of Merton's model outperforms a simple version of the traditional implementation of the model. It is reassuring that it also outperforms an alternative way of deriving credit spreads from implied volatilities that is based on a model with less structure.

Two predictions made by Merton's model are:

- ❑ there should be a positive relationship with positive convexity between credit spreads and at-the-money volatilities; and
- ❑ there should be a positive relationship between credit spreads and volatility skews when the at-the-money volatility is high.

The first prediction is strongly supported by the data. The second is also supported by the data but somewhat less strongly than the first.

APPENDIX

Kendall and Gibbons (1990) provide a great deal of information on the statistical properties of the Kendall and Spearman rank correlation measures. For $n > 10$, the probability distribution of Kendall's rank order correlation, r_k, conditional on no rank order correlation between the variables is approximately normal with a mean of zero and a variance of $[2(2n + 5)]/[9n(n - 1)]$. The z-statistic for testing the null hypothesis that r_k is zero is therefore

$$\frac{3r_k\sqrt{n(n-1)}}{\sqrt{2(2n+5)}}$$

For $n > 30$ the probability distribution of Spearman's rank order correlation, r_s, conditional on no rank order correlation is approximately normal with a mean of zero and a variance of $1/(n - 1)$. The z-statistic for testing the null hypothesis that r_s is zero is therefore $r_s\sqrt{n-1}$.

When the rank order correlation is non-zero, Kendall and Gibbons show that the standard deviation of the estimate of r_k depends on the true value of r_k and other unknown quantities concerned with the arrangement of the ranks in the parent population. The same is true of r_s. The estimated value of r_k can be assumed to be drawn from a normal distribution with a mean of ρ_k and a variance of at most $2(1 - \rho_k^2)/n$, where ρ_k is the true Kendall rank order correlation. The estimated value of r_s can be assumed to be drawn from a normal distribution with a mean of ρ_s and a variance of at most $3(1 - \rho_s^2)/n$, where ρ_s is the true Spearman's rank order correlation. In practice, ρ_k and ρ_s are set equal to the estimates, r_k and r_s, in these formulas.

These results enable us to construct a conservative test of whether there is a significant difference between two rank order correlations. For example, suppose we observe a Spearman rank order correlation of $r_{s,1}$ from a sample of n_1 and a Spearman rank order correlation of $r_{s,2}$ from a sample of n_2. The z-statistic for testing whether they are significantly different is

$$\frac{r_{s,1} - r_{s,2}}{\sqrt{3\left(1 - r_{s,1}^2\right)/n_1 + 3\left(1 - r_{s,2}^2\right)/n_2}} \qquad \text{(A1)}$$

Suppose that $r_{k,j}$ and $r_{s,j}$ are the Kendall rank order correlation and Spearman rank order correlation for company j, that n_j is the number of observations for company j, and that there are N companies. Under the null hypothesis that there is no correlation between the two variables, each of the $r_{k,j}$ is normally distributed with mean zero and variance

$$V_j = \frac{2(2n_j + 5)}{\left(9n_j(n_j - 1)\right)}$$

The mean value of the $r_{k,j}$ is then normally distributed with mean zero and variance

$$\frac{1}{N^2}\sum_{j=1}^{N} V_j$$

and the z-statistic for testing whether the mean is significantly different from zero is

$$z_k = \frac{\sum_{j=1}^{N} r_{k,j}}{\sqrt{\sum_{j=1}^{N} 2(2n_j + 5)\big/\left(9n_j(n_j - 1)\right)}}$$

An upper bound for the standard error of each of the $r_{k,j}$ is $2(1 - r_{k,j}^2)/n_j$ and the standard error of the mean value of the $r_{k,j}$ is

$$\frac{1}{N}\sqrt{\sum_{j=1}^{N} 2\left(1 - r_{k,j}^2\right)\big/n_j}$$

Analogously, the z-statistic for the mean value of the $r_{s,j}$ is

$$z_s = \frac{\sum_{j=1}^{N} r_{s,j}}{\sqrt{\sum_{j=1}^{N} \frac{1}{n_j - 1}}}$$

and an upper bound for the standard error of the mean value of the $r_{s,j}$ is

$$\frac{1}{N}\sqrt{\sum_{j=1}^{N} 3\left(1 - r_{s,j}^2\right)\big/n_j}$$

The expressions for the z-statistics and standard error for daily means in the day-by-day analysis are similar to those in the company-by-company analysis. In this case, j counts days rather than companies and N is the number of days for which we are able to calculate rank order correlations.

References

Black, F., and Cox, J. (1976). Valuing corporate securities: some effects of bond indenture provisions. *Journal of Finance* **31**, 351–67.

Black, F., and Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy* **81**, 637–59.

Blanco, R. S., Brennan, S., and Marsh, I. W. (2003). An empirical analysis of the dynamic relationship between investment grade bonds and credit default swaps. Forthcoming in *Journal of Finance.*

Collin-Dufresne, P., and Goldstein, R. (2001). Do credit spreads reflect stationary leverage ratios? *Journal of Finance* **52**, 1929–57.

Duffie, D. (1999). Credit swap valuation. *Financial Analysts Journal* **55**, 73–89.

Duffie, D., and Singleton, K. (1999). Modeling term structures of defaultable bonds. *Review of Financial Studies* **12**, 687–720.

Eom, Y. H., Helwege, J., and Huang, J.-Z. (2004). Structural models of corporate bond pricing: an empirical analysis. *Review of Financial Studies* **17**, 499–544.

Finger, C. (2002). CreditGrades technical document. www.creditgrades.com (May).

Gemmill, G. (2002). Testing Merton's model for credit spreads on zero-coupon bonds. Working paper, City University Business School, London.

Geske, R. (1977). The valuation of corporate liabilities as compound options. *Journal of Financial and Quantitative Analysis* **5**, 541–52.

Geske, R. (1979). The valuation of compound options. *Journal of Financial Economics* **7**, 63–82.

Hull, J., Predescu, M., and White, A. (2004). The relationship between credit default swap spreads, bond yields, and credit ratings announcements. *Journal of Banking and Finance* **28**, 2789–811.

Hull, J., and White, A. (2000). Valuing credit default swaps I: no counterparty default risk. *Journal of Derivatives* **8**, 29–40.

Jackwerth, J. C., and Rubinstein, M. (1996). Recovering probabilities from option prices. *Journal of Finance* **51**, 1611–31.

Jones, E. P., Mason, S. P., and Rosenfeld, E. (1984). Contingent claims analysis of corporate capital structure: an empirical investigation. *Journal of Finance* **39**, 611–25.

Kealhofer, S. (2003a). Quantifying credit risk I: default prediction. *Financial Analysts Journal* **59**(1), 30–44.

Kealhofer, S. (2003b). Quantifying credit risk II: debt valuation. *Financial Analysts Journal* **59**(3), 78–92.

Kendall, M., and Gibbons, J. D. (1990). *Rank correlation methods*, Fifth edition. Edward Arnold, London.

Leland, H. E., and Toft, K. B. (1996). Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance* **51**, 987–1019.

Litterman, R., and Iben, T. (1991). Corporate bond valuation and the term structure of credit spreads. *Journal of Portfolio Management* **17**, 52–64.

Longstaff, F. A., and Schwartz, E. (1995). A simple approach to valuing risky fixed and floating debt. *Journal of Finance* **50**, 789–819.

Longstaff, F. A., Mithal, S., and Neis, E. (2003). Corporate yield spreads: default risk or liquidity? New evidence from the credit default swap market. Forthcoming in *Journal of Finance*.

Merton, R. C. (1974). On the pricing of corporate debt: the risk structure of interest rates. *Journal of Finance* **29**, 449–70.

Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics* **3**, 125–44.

Rodriguez, R. J. (1988). Default risk, yield spreads, and time to maturity. *Journal of Financial and Quantitative Analysis* **23**, 111–17.

Rubinstein, M. (1994). Implied binomial trees. *Journal of Finance* **49**, 771–818.